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2008 J. Phys. A: Math. Theor. 41 164053

(http://iopscience.iop.org/1751-8121/41/16/164053)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 164053 (11pp) doi:10.1088/1751-8113/41/16/164053

Dark energy and the fifth force problem

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Received 21 October 2007
Published 9 April 2008
Online at stacks.iop.org/JPhysA/41/164053

Abstract
Generally accepted explanation of the observed accelerated expansion of the present day universe is based on the idea of the existence of a new entity called dark energy. Resolution of the 'cosmic coincidence' problem implies that dark energy and dark matter follow the same scaling solution during a significant period of evolution. This becomes possible only if there exists a coupling of the dark energy (modeled by a light scalar field) to dark matter. This conclusion following from the observed cosmological data serves for an additional evidence of well-known theoretical predictions of a light scalar coupled to matter. However, according to the results of the fifth force experiments, a similar coupling of the light scalar field to visible (baryonic) matter is strongly suppressed. After a brief review of some models intended for resolution of this 'fifth force problem', we present a model with spontaneously broken scale invariance where the strength of the dilaton-to-matter coupling appears to be dependent on the matter density. This is realized without any special assumptions in the underlying action intended for obtaining such a dependence. As a result the dilaton-to-matter coupling constant measured under conditions of all known fifth force experiments turns out automatically (without any sort of fine tuning) to be so small that, at least in the near future, experiments will not be able to reveal it. On the other hand, if the matter is very diluted (such as galaxy halo dark matter) then its coupling to the dilaton may not be weak. However, the latter situation is realized under conditions not compatible with the design of the fifth force experiments.

PACS numbers: 04.50.Kd, 04.80.Cc, 95.36.+x

1. The fifth force problem in the light of present day cosmology

The acceleration of the spatially flat Friedmann–Robertson–Walker (FRW) universe is described by the equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p) \tag{1}$$

where $a(t)$ is the scale factor; dots mark derivative with respect to cosmic time; ρ and p are, respectively, the total energy density and pressure. The last decade's observational evidence for the accelerated expansion of the present day universe means that the total pressure must be negative $p < -\rho/3$, and that is incompatible with properties of both baryonic matter and dark matter. This is why, if we believe that General Relativity (GR) is applicable to cosmology, one should put forward the idea that the dominant component filling the present day universe has negative pressure. There is a well-known example of such an entity: the cosmological constant (CC) Λ having the sense of the vacuum energy density $\Lambda \equiv \rho_{\text{vac}}$ with the equation of state $p_{\text{vac}} = -\rho_{\text{vac}}$. It is evident, however, that CC could not be the same constant in different cosmological epochs since, for example, vacuum energy density changes in the course of phase transitions; the ratio of values of the vacuum energy density in the early inflation and in the present cosmological epoch is a huge number $\sim 10^{110} - 10^{120}$. The new entity responsible for the acceleration of the present day universe was given the name dark energy.

Analysis of observations yields a doubtless conclusion that dark energy density is about two times bigger than matter density in the present cosmological epoch. This appears an amazing fact indeed because even the order of magnitude of the densities of those two components, i.e. dark energy (which in a certain sense is a synonym of the term 'vacuum') and matter (really one can speak here about a dark matter which constitutes 96% of all matter), should be completely unrelated from the point of view of our knowledge in field theory and cosmology; note that these densities scale with time in a very different way. The question arises: why they are so close in the present cosmological epoch? This problem of 'why just now' has obtained the name 'cosmic coincidence' [1]. It is very hard to believe that this coincidence is accidental because it requires a double fine tuning both of the present extremely small vacuum energy density and of the matter density. If it is not an accidental coincidence but rather is a characteristic feature during a long enough period of evolution, then the explanation of this phenomenon suggests that there exists an exchange of energy between dark matter and dark energy.

The most developed and successful class of models intended to describe the dark energy uses a scalar field. The latter can mimic the dynamics governed by the present day cosmological constant if its potential $V(\phi)$ is very flat and its mass m is of the order of the present value of the Hubble parameter H_0:

$$\frac{\mathrm{d}^2 V}{\mathrm{d}\phi^2} = m^2 \sim H_0^2 \sim (10^{-33}\,\mathrm{eV})^2; \tag{2}$$

one can list for example the following models: cosmon, quintessence, coupled quintessence, Chaplygin gas, phantom, k-essence, etc; for a review see [2] and references therein. In the context of such models the exchange of energy between dark matter and dark energy occurs due to a direct scalar field-to-matter coupling.

The idea of the existence of a light scalar coupled to matter has a well-known theoretical ground, for example in string theory, in models with spontaneously broken dilatation symmetry [3, 4], in theories with extra dimensions. Coupling of the matter to a light scalar field inevitably produces a long-range scalar force. It is well known since the appearance of the Brans–Dicke model that such a 'fifth' force could affect the results of tests of GR and, in particular, may entail a violation of Einstein's equivalence principle. In the non-relativistic limit, the correction to the Newton law can be parametrized by a Yukawa-type correction

$$V(r) = -G\frac{m_1 m_2}{r}(1 + \beta\,\mathrm{e}^{-mr}) \tag{3}$$

A light scalar particle interacting with matter could also give rise to testable consequences in an intermediate, submillimeter or even shorter range depending on the scalar mass.

Numerous specially designed experiments lasting many years have not so far revealed any possible manifestations of the fifth force. This fact, on each stage of the sequence of experiments, is treated as a new, stronger constraint on the parameters (like coupling constant and mass) with hope that the next generation of experiments will be able to discover a scalar force modifying Newtonian gravity. This discrepancy between theory and experiment constitutes the essence of the fifth force problem in the 'narrow sense'[1]. A new aspect introduced by modern cosmology to this problem is the question of why the coupling of the light scalar (dark energy) to visible matter is strongly suppressed while similar coupling to dark matter is energetic. Discovery of dark energy and cosmic coincidence interpreted as evidence of the existence of an unsuppressed dark energy to dark matter coupling, turns the fifth force problem into an actual and even burning fundamental puzzle.

One interesting approach to the resolution of the fifth force problem, known since 1994 as 'the least coupling principle', is based on the idea of [5] to use non-perturbative string loop effects to explain why the massless dilaton may decouple from matter. In fact it was shown that *under certain assumptions* about the structure of the (unknown) dilaton coupling functions in the low-energy effective action resulting from taking into account the full non-perturbative string loop expansion, the string dilaton is cosmologically attracted toward values where its effective coupling to matter disappears.

The astrophysical effects of the matter density dependence of the dilaton to matter coupling was studied in 1989 in the context of a model with spontaneously broken dilatation symmetry in [4]. However, in this model the effect is too weak to be observed now.

Another way to describe the influence of the matter density on the fifth force is used in the Chameleon model [6] formulated in 2004. The key point here is the fact that the scalar field effective potential depends on the local matter density ρ_m if the direct coupling of the scalar field to the metric tensor in the underlying Lagrangian is assumed like in earlier models [7]. Therefore, the position of the minimum of the effective potential and the mass of small fluctuations turn out to be ρ_m dependent. In space regions of 'high' matter density such as on the Earth or in other compact objects, the effective mass of the scalar field becomes so big that the scalar field can penetrate only into a thin superficial shell of the compact object. As a result of this, it appears to be possible to realize a situation where in spite of a choice for a scalar to matter coupling of order unity, the violation of the equivalence principle is exponentially suppressed. However, for objects of lower density, the fifth force may be detectable and the corresponding predictions are made; see however results of the recent experiments [8, 9]. Besides, the following question arises if one considers a possible contribution of the chameleon scalar to the Casimir vacuum energy. In fact, the chameleon scalar is practically massless in the vacuum between two conducting plates. Taking into account that the coupling constant of its interaction with the matter of the plates is assumed of order unity, should it contribute to the Casimir vacuum energy? If yes, how can it happen that this contribution does not alter the well-established accordance between experimental and theoretical results taking into account contribution to the Casimir vacuum energy only from the electromagnetic vacuum? Does it mean that the chameleon model may be ruled out already by the existing Casimir vacuum energy data?

One should note that the model of [10] with the matter density dependence of the effective dilaton to matter coupling was constructed in 2001 without any specific conjectures in the underlying action intended to solve the fifth force problem. *The resolution of the fifth force problem appears as a result* which reads as follows. (1) The local effective Yukawa coupling of

[1] As is well known, other implications of the light scalar generically may be for cosmological variations of the vacuum expectation value of the Higgs field, the fine structure constant and other gauge coupling constants. However, in this paper we study only the strength of the fifth force itself.

the dilaton to fermions in normal laboratory conditions equals practically zero automatically, without any fine tuning of the parameters. The term 'normal laboratory conditions' means that the local fermion energy density is many orders of magnitude larger than the dilaton contribution to the dark energy density. (2) Under the same conditions, the Einstein's GR is reproduced. The opposite situation may be realized [11] if the matter is diluted up to a local energy density comparable with the dilaton contribution to the dark energy density (in this case we say that the matter is in the state of cosmo-low-energy physics (CLEP)). In the CLEP state a very interesting effect of the 'neutrino dark energy' appears [11]; besides one can show that the present dark energy may be dominated by the cosmological constant Λ and that the transition from the light dilaton dominated stage to the Λ dominated stage was about 10 billion years ago. Nevertheless the light dilaton still could have local effects in the present astronomy and Casimir experiments. However, under conditions of these experiments, the dilaton practically decouples from the visible matter.

In the present paper, we demonstrate that the mechanism of resolution of the fifth force problem studied in models [10, 11], operates in a similar way in a more general case, namely when using a macroscopic description of the matter.

2. Main ideas of the two measures field theory

TMT is a generally coordinate invariant theory where *all the difference from the standard field theory in curved spacetime consists only of the following three additional assumptions*.

(1) The first assumption is the hypothesis that the effective action at the energies below the Planck scale has to be of the form [12–20]

$$S = \int L_1 \Phi \, d^4x + \int L_2 \sqrt{-g} \, d^4x \tag{4}$$

including two Lagrangians L_1 and L_2 and two measures of integration $\sqrt{-g}$ and Φ. One is the usual measure of integration $\sqrt{-g}$ in the four-dimensional spacetime manifold equipped with the metric $g_{\mu\nu}$. Another is the new measure of integration Φ in the same four-dimensional spacetime manifold. The measure Φ being a scalar density and a total derivative may be defined for example by means of four scalar fields φ_a $(a = 1, 2, 3, 4)$

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \tag{5}$$

(2) Generically it is allowed that L_1 and L_2 will be functions of all matter fields, the dilaton field, the metric, the connection but not of the 'measure fields' φ_a. In such a case, i.e. when the measure fields enter in the theory only via the measure Φ, the action (4) possesses an infinite dimensional symmetry $\varphi_a \to \varphi_a + f_a(L_1)$, where $f_a(L_1)$ are arbitrary functions of L_1 (see details in [14]). One can hope that this symmetry should prevent emergence of a measure field dependence in L_1 and L_2 after quantum effects are taken into account.

(3) Important feature of TMT that is responsible for many interesting and desirable results of the field theory models studied so far [10–20] consists of the assumption that all fields are independent dynamical variables. All the relations between them are results of equations of motion. In particular, the independence of the metric and the connection means that we proceed in the first-order formalism and the relation between the connection and metric is not *a priori* according to the Riemannian geometry.

We want to stress again that except for the three listed assumptions, we do not make any changes as compared with principles of the standard field theory in curved spacetime. In other words, all the freedom in constructing different models in the framework of TMT consists of the choice of the concrete matter content and the Lagrangian densities L_1 and L_2 that is quite similar to the standard field theory.

Since Φ is a total derivative, a shift of L_1 by a constant, $L_1 \to L_1 + \text{const}$, has no effect on the equations of motion. Similar shift of L_2 would lead to the change of the constant part of the Lagrangian coupled to the volume element $\sqrt{-g}\, \mathrm{d}^4x$. In the standard GR, this constant term is the cosmological constant. However, in TMT the relation between the constant term of L_2 and the physical cosmological constant results in such a form that makes possible [10, 11, 14, 19, 20] to resolve the cosmological constant problem.

Variation of the measure fields φ_a yields $B^\mu_a \partial_\mu L_1 = 0$ where $B^\mu_a = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d$. Since $\mathrm{Det}\left(B^\mu_a\right) = \frac{4^{-4}}{4!}\Phi^3$ it follows that if $\Phi \neq 0$,

$$L_1 = sM^4 = \text{const}, \tag{6}$$

where $s = \pm 1$ and M is a constant of integration with the dimensions of mass. In what follows we make the choice $s = 1$.

Applying the Palatini formalism in TMT one can show (see for example [14]) that in addition to the usual Christoffel coefficients, the resulting relation between metric and connection includes also the gradient of the ratio of the two measures

$$\zeta \equiv \frac{\Phi}{\sqrt{-g}} \tag{7}$$

which is a scalar field. This means that with the set of variables used in the underlying action (4) (and in particular with the metric $g_{\mu\nu}$) the spacetime is not Riemannian. The gravity and matter field equations obtained by means of the first-order formalism contain both ζ and its gradient. It turns out that at least at the classical level, the measure fields φ_a affect the theory only through the scalar field ζ.

Variation with respect to the metric yields as usual the gravitational equations. If L_1 involves a scalar curvature term (or other curvature invariants) then equation (6) provides us with an additional gravitational-type equation, independent of the former. Taking trace of the gravitational equations and excluding the scalar curvature from these independent equations, we obtain a consistency condition having the form of a constraint which determines $\zeta(x)$ as a function of matter fields. It is very important that neither Newton constant nor curvature appears in this constraint which means that the *geometrical scalar field* $\zeta(x)$ *is determined by other fields configuration* locally and straightforward (that is without gravitational interaction).

By an appropriate change of the dynamical variables which includes a redefinition of the metric, one can formulate the theory in a Riemannian spacetime. The corresponding frame we call 'the Einstein frame'. The big advantage of TMT is that in a very wide class of models, *the gravity and all matter field equations of motion take canonical GR form in the Einstein frame*. All the novelty of TMT in the Einstein frame as compared with the standard GR is revealed only in an unusual structure of the scalar field effective potential, masses of particles and their interactions with scalar fields as well as in the unusual structure of matter contributions to the energy–momentum tensor: all these quantities appear to be ζ dependent. This is why the scalar field $\zeta(x)$ determined by the constraint as a function of matter fields, has a key role in dynamics of TMT models.

3. Scale invariant model

In the original frame (where the metric is $g_{\mu\nu}$), a matter content of our TMT model represented in the form of the action (4), is a dust and a scalar field (dilaton). The dilaton ϕ allows us to realize a spontaneously broken global scale invariance [10, 11, 15–18] and together with this it can govern the evolution of the universe on different stages: in the early universe ϕ plays the role of inflaton and in the late time universe it is transformed into a part of the dark energy (for details, see [10, 11, 19]). We postulate that the theory is invariant under the global scale transformations.

$$g_{\mu\nu} \to \mathrm{e}^{\theta} g_{\mu\nu}, \qquad \Gamma^{\mu}_{\alpha\beta} \to \Gamma^{\mu}_{\alpha\beta}, \qquad \phi \to \phi - \frac{M_p}{\alpha}\theta, \qquad \varphi_a \to l_{ab}\varphi_b \tag{8}$$

where $\det(l_{ab}) = \mathrm{e}^{2\theta}$ and $\theta = \text{const}$. Keeping the general structure (4), it is convenient to represent the action in the following form:

$$\begin{aligned} &S = S_g + S_\phi + S_m \\ &S_g = -\frac{1}{\kappa}\int (\Phi + b_g\sqrt{-g}) R(\Gamma, g)\, \mathrm{e}^{\alpha\phi/M_p}\, \mathrm{d}^4 x; \\ &S_\phi = \int \mathrm{e}^{\alpha\phi/M_p}\left[(\Phi + b_\phi\sqrt{-g})\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - (\Phi V_1 + \sqrt{-g}V_2)\,\mathrm{e}^{\alpha\phi/M_p}\right]\mathrm{d}^4 x; \\ &S_m = \int (\Phi + b_m\sqrt{-g}) L_m\, \mathrm{d}^4 x, \end{aligned} \tag{9}$$

where $R(\Gamma, g) = g^{\mu\nu}\left(\Gamma^{\lambda}_{\mu\nu,\lambda} + \Gamma^{\lambda}_{\alpha\lambda}\Gamma^{\alpha}_{\mu\nu} - (\nu \leftrightarrow \lambda)\right)$, $M_p = (8\pi G)^{-1/2}$ and the Lagrangian for the matter, as collection of particles, which provides the scale invariance of S_m reads

$$L_m = -m\sum_i \int \mathrm{e}^{\frac{1}{2}\alpha\phi/M_p}\sqrt{g_{\alpha\beta}\frac{\mathrm{d}x_i^{\alpha}}{\mathrm{d}\lambda}\frac{\mathrm{d}x_i^{\beta}}{\mathrm{d}\lambda}}\,\frac{\delta^4(x - x_i(\lambda))}{\sqrt{-g}}\,\mathrm{d}\lambda \tag{10}$$

where λ is an arbitrary parameter. For simplicity, we consider the collection of the particles with the same mass parameter m. We assume in addition that $x_i(\lambda)$ do not participate in the scale transformations (8).

In the action (9), there are two types of the gravitational terms and of the 'kinetic-like terms' which respect the scale invariance: the terms of the one type coupled to the measure Φ and those of the other type coupled to the measure $\sqrt{-g}$. Using the freedom in normalization of the measure fields φ_a, we set the coupling constant of the scalar curvature to the measure Φ to be $-\frac{1}{\kappa}$, where $\kappa = 16\pi G$. Normalizing all the fields such that their couplings to the measure Φ have no additional factors, we are not in general able to provide the same in terms describing the appropriate couplings to the measure $\sqrt{-g}$. This fact explains the need to introduce the dimensionless real parameters b_g, b_ϕ and b_m. We will only assume that they are positive, have the same or very close orders of magnitude

$$b_g \sim b_\phi \sim b_m \tag{11}$$

and besides $b_m > b_g$. The real parameter $\alpha > 0$ is assumed to be of the order of unity.

One should also point out the possibility of introducing two different pre-potentials which are exponential functions of the dilaton ϕ coupled to the measures Φ and $\sqrt{-g}$ with factors V_1 and V_2. Such ϕ-dependence provides the scale symmetry (8). We will see below how the dilaton effective potential is generated as the result of SSB of the scale invariance and a transformation to the Einstein frame.

According to the general prescriptions of TMT, we have to start from studying the self-consistent system of gravity (metric $g_{\mu\nu}$ and connection $\Gamma^{\mu}_{\alpha\beta}$), the measure Φ degrees of freedom φ_a, the dilaton field ϕ and the matter particle coordinates $x_i^{\alpha}(\lambda)$, proceeding in the first-order formalism.

For the purpose of this paper, we restrict ourselves to a zero temperature gas of particles, i.e. we will assume that $\mathrm{d}\vec{x}_i/\mathrm{d}\lambda \equiv 0$ for all particles. It is convenient to proceed in the frame where $g_{0l} = 0, l = 1, 2, 3$. Then the particle density is defined by

$$n(\vec{x}) = \sum_i \frac{1}{\sqrt{-g_{(3)}}}\delta^{(3)}(\vec{x} - \vec{x}_i(\lambda)) \tag{12}$$

where $g_{(3)} = \det(g_{kl})$ and

$$S_m = -m \int \mathrm{d}^4x(\Phi + b_m\sqrt{-g})n(\vec{x})\,\mathrm{e}^{\frac{1}{2}\alpha\phi/M_p}. \tag{13}$$

Following the procedure described in the previous section we have to write down all equations of motion, find the consistency condition (which determines ζ-field as a function of other fields and matter) and transform to the Einstein frame. Note that with the action (9), equation (6) describes a *spontaneous breakdown of the global scale symmetry* (8). It turns out that when working with the new metric

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{\alpha\phi/M_p}(\zeta + b_g)g_{\mu\nu}, \tag{14}$$

which we call the Einstein frame, the connection becomes Riemannian. Since $\tilde{g}_{\mu\nu}$ is invariant under the scale transformations (8), spontaneous breaking of the scale symmetry (by means of equation (6)) is reduced in the Einstein frame to the *spontaneous breakdown of the shift symmetry* $\phi \to \phi + \mathrm{const}$. Note that the Goldstone theorem generically is not applicable in this kind of models [18].

The transformation (14) causes the transformation of the particle density

$$\tilde{n}(\vec{x}) = (\zeta + b_g)^{-3/2}\,\mathrm{e}^{-\frac{3}{2}\alpha\phi/M_p}n(\vec{x}). \tag{15}$$

After the change of variables to the Einstein frame (14) and some simple algebra, the gravitational equations take the standard GR form

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2}T^{\mathrm{eff}}_{\mu\nu} \tag{16}$$

where $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian spacetime with the metric $\tilde{g}_{\mu\nu}$. The components of the effective energy–momentum tensor are as follows:

$$T^{\mathrm{eff}}_{00} = \frac{\zeta + b_\phi}{\zeta + b_g}(\dot{\phi}^2 - \tilde{g}_{00}X) + \tilde{g}_{00}\left[V_{\mathrm{eff}}(\phi;\zeta,M) - \frac{\delta\cdot b_g}{\zeta + b_g}X + \frac{3\zeta + b_m + 2b_g}{2\sqrt{\zeta + b_g}}m\tilde{n}\right] \tag{17}$$

$$T^{\mathrm{eff}}_{ij} = \frac{\zeta + b_\phi}{\zeta + b_g}(\phi_{,k}\phi_{,l} - \tilde{g}_{kl}X) + \tilde{g}_{kl}\left[V_{\mathrm{eff}}(\phi;\zeta,M) - \frac{\delta\cdot b_g}{\zeta + b_g}X + \frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}}m\tilde{n}\right]. \tag{18}$$

Here we use the notations $X \equiv \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$, $\delta = (b_g - b_\phi)/b_g$ and

$$V_{\mathrm{eff}}(\phi;\zeta) = \frac{b_g[M^4\,\mathrm{e}^{-2\alpha\phi/M_p} + V_1] - V_2}{(\zeta + b_g)^2}. \tag{19}$$

The dilaton ϕ field equation in the Einstein frame is as follows

$$\begin{aligned}\frac{1}{\sqrt{-\tilde{g}}}\partial_\mu &\left[\frac{\zeta + b_\phi}{\zeta + b_g}\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right] \\ &- \frac{\alpha}{M_p}\frac{(\zeta + b_g)M^4\,\mathrm{e}^{-2\alpha\phi/M_p} - (\zeta - b_g)V_1 - 2V_2 - \delta b_g(\zeta + b_g)X}{(\zeta + b_g)^2} \\ &= \frac{\alpha}{M_p}\frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}}m\tilde{n}.\end{aligned} \tag{20}$$

The scalar field ζ in equations (17)–(20) is determined as a function $\zeta(\phi, X, \tilde{n})$ by means of the following consistency condition:

$$\frac{(b_g-\zeta)(M^4\,\mathrm{e}^{-2\alpha\phi/M_p}+V_1)-2V_2}{(\zeta+b_g)^2}-\frac{\delta\cdot b_g X}{\zeta+b_g}=\frac{\zeta-b_m+2b_g}{2\sqrt{\zeta+b_g}}m\tilde{n}. \tag{21}$$

One should point out two very important features of the model. First, the ϕ dependence in all the equations of motion (including the constraint) emerges only in the form $M^4\,\mathrm{e}^{-2\alpha\phi/M_p}$ where M is the integration constant, i.e. due to the spontaneous breakdown of the scale symmetry (8) (or the shift symmetry $\phi \rightarrow \phi + \text{const.}$ in the Einstein frame). Second, equation (21) is the fifth-degree algebraic equation with respect to $\sqrt{\zeta+b_g}$ and therefore generically ζ is a complicated function of ϕ, X and $\tilde{n}$. Hence, generically each of ζ dependent terms in equations (17)–(20) describe very nontrivial coupling of the dilaton to the matter.

4. Dark energy in the absence of matter

In the simplest case, when the particle density of the dust is zero: $\tilde{n}(x) \equiv 0$ the dilaton ϕ is the only matter which in the early universe plays the role of the inflaton while in the late universe it is the dark energy. The appropriate model in the context of cosmological solutions has been studied in detail in [19]. Here, we present only some of the equations we will need for the purposes of this paper and a list of the main results.

In the absence of the matter particles, the scalar $\zeta = \zeta(\phi, X)$ can easily be found from the consistency condition (21). In the spatially homogeneous case $X \geqslant 0$ (we use the signature $(+---)$). Then the effective energy–momentum tensor can be represented in a form of that of a perfect fluid $T^{\mathrm{eff}}_{\mu\nu} = (\rho+p)u_\mu u_\nu - p\tilde{g}_{\mu\nu}$, where $u_\mu = \phi_{,\mu}/\sqrt{2X}$ with the following energy and pressure densities where now $\tilde{n}(x) \equiv 0$

$$\rho(\phi, X; M) \equiv \rho^{(\tilde{n}=0)} = X + \frac{(M^4\,\mathrm{e}^{-2\alpha\phi/M_p}+V_1)^2 - 2\delta b_g(M^4\,\mathrm{e}^{-2\alpha\phi/M_p}+V_1)X - 3\delta^2 b_g^2 X^2}{4[b_g(M^4\,\mathrm{e}^{-2\alpha\phi/M_p}+V_1)-V_2]}, \tag{22}$$

$$p(\phi, X; M) \equiv p^{(\tilde{n}=0)} = X - \frac{(M^4\,\mathrm{e}^{-2\alpha\phi/M_p}+V_1+\delta b_g X)^2}{4[b_g(M^4\,\mathrm{e}^{-2\alpha\phi/M_p}+V_1)-V_2]}. \tag{23}$$

Substitution of $\zeta(\phi, X)$ into the ϕ equation yields the appearance of the nonlinear X dependence. This means that in spite of the absence of such nonlinearity in the underlying action, our model represents an explicit example of k-essence [21] resulting from first principles. The effective k-essence action is as follows:

$$S_{\mathrm{eff}} = \int \sqrt{-\tilde{g}}\,\mathrm{d}^4x\left[-\frac{1}{\kappa}R(\tilde{g}) + p(\phi, X; M)\right]. \tag{24}$$

In the context of spatially flat FRW cosmology, in the absence of the matter particles (i.e $\tilde{n}(x) \equiv 0$), the TMT model under consideration exhibits a number of possible outputs [19] depending on the choice of regions in the parameter space (but without fine tuning).

(a) *Absence of initial singularity of the curvature while its time derivative is singular*. This is a sort of 'sudden' singularities studied by Barrow [22].
(b) Power law inflation in the subsequent stage of evolution with a *graceful exit* from inflation.
(c) Possibility of *resolution of the old CC problem*.
(d) TMT enables us to achieve small CC without fine tuning of dimensionful parameters.

(e) There is a wide range of parameters where the dynamics of the scalar field ϕ, playing the role of the dark energy in the late universe, allow crossing the phantom divide, i.e. the equation-of-state $w = p/\rho$ may be $w < -1$ and w asymptotically (as $t \to \infty$) approaches -1 from below.

Taking into account that in the late time universe the X-contribution to $\rho|_{\tilde{n}=0}$ approaches zero, one can see that the dark energy density is positive for any ϕ provided

$$b_g V_1 \geqslant V_2. \tag{25}$$

Then it follows from (21) as $\tilde{n} = 0$ that

$$|\zeta^{(\tilde{n}=0)}| \sim b_g. \tag{26}$$

This will be useful in the next section.

5. Normal conditions: reproducing Einstein's GR and absence of the fifth force problem

One should now pay attention to the interesting result that the explicit $\tilde{n}$ dependence involving **the same form of ζ dependence**

$$\frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} m\tilde{n} \tag{27}$$

appears simultaneously[2] in the dust contribution to the pressure (through the last term in equation (18)), in the effective dilaton to dust coupling (in the r.h.s. of equation (20)) and in the r.h.s. of the consistency condition (21).

Let us analyze the consequences of this wonderful coincidence in the case when matter energy density (modeled by dust) is many orders of magnitude larger than the dilaton contribution to the dark energy density in the space region occupied by this matter. Evidently this is the condition under which all tests of Einstein's GR, including measurements of the fifth force, are fulfilled. Therefore if this condition is satisfied we will say that the matter is in **normal conditions**. The existence of the fifth force turns into a problem just in normal conditions that is a key point which allows to resolve it.

The detailed analysis of the equations of motion together with the consistency condition (21) yields the result (see [23]) that in normal conditions (n.c.) the following equality holds with extremely high accuracy:

$$\zeta^{(n.c.)} \approx b_m - 2b_g. \tag{28}$$

Recall that we have assumed $b_m > b_g$. Then $\zeta^{(n.c.)} + b_g > 0$, and the transformation (14) and the subsequent equations in the Einstein frame are well defined. Taking into account our assumption (11) and equation (26) we infer that $\zeta^{(n.c.)}$ and $\zeta^{(\tilde{n}=0)}$ have close orders of magnitudes. Then it is easy to see (making use the inequality (25)) that the l.h.s. of equation (21), as $\zeta = \zeta^{(n.c.)}$, has the order of magnitude close to that of the dark energy density $\rho^{(\tilde{n}=0)}$ in the absence of matter case discussed in section 4. Thus in the case under consideration, *the consistency condition* (21) *describes a balance between the pressure of the dust in normal conditions on the one hand and the vacuum energy density on the other hand* [3].

The last terms in equations (17) and (18), being the dust contributions to the energy density (ρ_m) and the pressure ($-p_m$) respectively, generally speaking have the same order of

[2] Note that an analogous result has been observed earlier in the model [10, 11] where fermionic matter has been studied instead of the macroscopic (dust) matter in the present model.

[3] As a matter of fact, normal condition for dust means that the dust is not diluted, i.e. the particle density $\tilde{n}$ is large enough in comparison with the averaged cosmological or galaxy halo particle densities.

magnitude. But if the dust is in the normal conditions then it follows from equation (28) that the dust becomes practically pressureless as it must be in GR.

Inserting (28) into the last term of equation (17), we obtain the dust energy density in normal conditions

$$\rho_m^{(n.c.)} = 2\sqrt{b_m - b_g}\, m\tilde{n}. \tag{29}$$

Substitution of (28) into the rest of the terms of the components of the energy–momentum tensor (17) and (18) gives the dilaton contribution to the energy density and pressure of the dark energy which have the orders of magnitude close to those in the absence of matter case, equations (22) and (23). The latter statement may be easily checked by using our assumption (11), results of section 4 and equations (26) and (28).

Besides reproducing Einstein equations when the dilaton and dust (in normal conditions) are sources of the gravity, the condition (28) automatically provides a practical disappearance of the effective dilaton to matter coupling. This one can see immediately inserting (28) into the ϕ equation (20). Let us however estimate the Yukawa-type coupling constant in the r.h.s. of equation (20). In fact, using the consistency condition (21) and representing the particle density in the form $\tilde{n} \approx N/\upsilon$ where N is the number of particles in a volume υ, one can make the following estimation for the effective dilaton to matter coupling 'constant' f defined by the Yukawa-type interaction term $f\tilde{n}\phi$ (if we were to invent an effective action whose variation with respect to ϕ would result in equation (20)):

$$f \equiv \alpha \frac{m}{M_p} \frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} \approx \alpha \frac{m}{M_p} \frac{\zeta - b_m + 2b_g}{2\sqrt{b_m - b_g}} \sim \frac{\alpha}{M_p} \frac{\rho_{\rm vac}}{\tilde{n}} \approx \alpha \frac{\rho_{\rm vac}\upsilon}{N M_p}. \tag{30}$$

Thus, we conclude that *the effective dilaton to matter coupling 'constant' in the normal conditions is of the order of the ratio of the 'mass of the vacuum' in the volume occupied by the matter to the Planck mass taking N times*. In some sense this result resembles the *Archimedes law*. At the same time equation (30) gives us an estimation of the exactness of the condition (28).

Acknowledgments

One of us (AK) acknowledges the organizers of the Workshop QFEXT07 and especially M Bordag for a support. He also thanks V Mostepanenko and G Klimchitskaya for helpful conversations.

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